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Total chromatic number of honeycomb network. (English) Zbl 1487.05100

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Total coloring of a simple graph (i.e., loopless, undirected, and without multiple edges) is a proper coloring of its vertices as well as its edges: whenever two vertices are adjacent, two edges intersect, or a vertex lies on an edge, the respective two objects are given distinct colors. The least number of colors required for a total coloring of the graph G is called the total chromatic number of G and is denoted $\chi_T(G)$. The total coloring conjecture of *M. Behzad* [Graphs and their chromatic numbers. East Lansing, MI: Michigan State University (PhD Thesis) (1965), Conjecture 1] and *V. G. Vizing* [Russ. Math. Surv. 23, No. 6, 125–141 (1968; Zbl 0192.60502); translation from Usp. Mat. Nauk 23, No. 6(144), 117–134 (1968)] says that $\Delta(G)+1 \leq \chi_T(G) \leq \Delta(G)+2$ for any simple graph G, where $\Delta(G)$ denotes its maximum degree.

The honeycomb network $\mathrm{HC}(n)$ for $n \geq 1$ is defined inductively as follows. $\mathrm{HC}(1)$ is a hexagon, or a 6-cycle. For $n \geq 2$, $\mathrm{HC}(n)$ is obtained from $\mathrm{HC}(n-1)$ simply by adding $6(n-1)$ hexagons around the boundary of $\mathrm{HC}(n-1)$. One observes that $\Delta(\mathrm{HC}(n)) = 3$ for $n > 1$, and $\Delta(\mathrm{HC}(1)) = 2$.

In this short paper, the authors provide an explicit total coloring of $\mathrm{HC}(n)$ using $\Delta(\mathrm{HC}(n))+2$ colors, thereby demonstrating that $\chi_T(\mathrm{HC}(n)) \leq \Delta(\mathrm{HC}(n))+2$ for all $n \geq 1$. For $n \geq 2$, the coloring is described using the level-numbering scheme in [*A. Sharieh* et al., "Hex-cell: Modeling, topological properties and routing algorithm", Eur. J. Sci. Res. 22, No. 2, 457–468 (2008)]. Appropriately coloring the vertices in each line using the colors c_1, c_2, the edges in each line using c_3, c_4, and the edges between adjacent lines using c_5, a total coloring of $\mathrm{HC}(n)$ for $n \geq 2$ using 5 colors is found. For a total coloring of $\mathrm{HC}(1)$ using 4 colors, the vertices are colored using c_1, c_2, and the edges using c_3, c_4.

Since $\Delta(\mathrm{HC}(n)) \leq 3$ for all $n \geq 1$, the result proved in this paper follows from a more general result by *N. Vijayaditya* [J. Lond. Math. Soc., II. Ser. 3, 405–408 (1971; Zbl 0223.05103)] which says that the total coloring conjecture holds for graphs with maximum degree of at most three. However, Vijayaditya's method (Lemma 3) when applied directly to the honeycomb network gives a total coloring only by induction on the number of vertices. On the other hand, the method used by the authors here is similar in spirit to that of Lemmas 1 and 2 in Vijayaditya's paper [loc. cit.].

A typo in the present paper: Corollary 2.1.1 should say that $\chi_T(\mathrm{HC}(1)) \leq 4$ instead of $\chi_T(\mathrm{HC}(1)) = 4$. Indeed, since $\mathrm{HC}(1)$ is a 6-cycle, three colors suffice for a total coloring of this graph; more generally, a well-known result is that for a cycle C_n on n vertices, $\chi_T(C_n) = 3$ if $n \equiv 0 \pmod 3$, and $\chi_T(C_n) = 4$ otherwise.

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MSC:

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05C82 Small world graphs, complex networks (graph-theoretic aspects)
05C10 Planar graphs; geometric and topological aspects of graph theory

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